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Relationships between precipitation amounts and occurrence of precipitation in Iceland

I) Introduction

The aim of this short study is to explore the role played by the intermittency phenomena on the spatial and temporal distribution of precipitation in Iceland. First, a linear relationship between precipitation and intermittency is derived. Then the validity of this relationship is tested with monthly and annual precipitation. Finally, the spatial and temporal variability of precipitation is discussed in relation to the characteristics of the proposed model.

II) Relationships between precipitation amounts and occurrence of precipitation

Let R denote the instantaneous precipitation rate in mm/h. R is an intermittent process in both space and time (it rains/snows or not) and thus admits a mixed distribution with a spike at zero (probability that R is null) and a continuous distribution for the strictly positive values. From this mixed distribution, one can derive the mean precipitation rate at any location (x,y) as follows:

$$E[R] = E[R|R > 0] \Pr[R > 0]$$
(1)

and the probability that the precipitation rate exceeds a given threshold τ :

$$\Pr[R > \tau] = \Pr[R > 0] \Pr[R > \tau | R > 0]$$
(2)

or

$$\Pr[R > 0] = \frac{\Pr[R > \tau]}{\Pr[R > \tau \mid R > 0]}$$
(3)

from (1) and (3), it is straightforward to write

$$E[R] = \frac{E[R|R>0]\Pr[R>\tau]}{\Pr[R>\tau|R>0]}$$
(4)

or

$$E[R] = S(\tau) \Pr[R > \tau]$$
(5)

where

$$S(\tau) = \frac{E[R|R>0]}{\Pr[R>\tau|\ R>0]}$$
(6)

or

$$S(\tau) = \frac{E[R|R>0]}{1 - \Pr[R \le \tau | R>0]} \tag{7}$$

The relationship (5) is true for any mixed distribution. It can be seen as a generalization of (1). The coefficient $S(\tau)$ can be seen as a scaled value of E[R|R>0] in order to account for the precipitation rates below the threshold τ . If $\tau=0$, $S(\tau)=E[R|R>0]$.

Let P denote the accumulated precipitation over a period of T days:

$$P = \sum_{i} R_{i} \Delta t \tag{8}$$

$$P = TE[R] = TS(\tau)\Pr[R > \tau]$$
(9)

$$P = \beta(\tau) \Pr[R > \tau] \tag{10}$$

where

$$\beta(\tau) = TS(\tau) \tag{11}$$

In practice, the precipitation rate R is not measured instantaneously but averaged over a period Δt ranging from a few minutes to a day. The probability that R exceeds a given threshold τ within the period T, $\Pr[R > \tau]$ is not known precisely but estimated by the Fractional Time R is exceeding τ , $FTR(\tau)$. First a binary variable is defined:

$$I(\tau) = \begin{cases} 1 & \text{if } R > \tau \\ 0 & \text{otherwise} \end{cases}$$
 (12)

then

$$FTR(\tau) = \frac{1}{n} \sum_{k=1}^{n} I_k(\tau)$$
 (13)

where n is the number of discrete measurements within the period T. If n is large enough with respect to the accumulating period T,

$$\Pr[R > \tau] \approx FTR(\tau) \tag{14}$$

Finally, the precipitation amount P at location (x,y) is estimated by:

$$P = \beta(\tau)FTR(\tau) \tag{15}$$

The linear relationship (15) shows that the precipitation amount can be represented by the product of two quantities:

- $FTR(\tau)$ which defines "how often" precipitation is occurring above the threshold τ throughout the period T.
- $\beta(\tau)$ which defines "how much" precipitation, each time precipitation is occurring above the threshold τ .

A geometrical interpretation of (15) is given in figure 1.

The variability of the precipitation amounts in both space and time depends on both variabilities of $FTR(\tau)$ and $\beta(\tau)$ (i.e. $S(\tau)$).

The relationship (5) is known as the threshold method. Number of studies based on (5) applied in both space and time and in various climatic regions have shown that $S(\tau)$ is usually quite stationary in space and time for some optimal threshold τ . If this is the case, one only needs to estimate $Pr(R > \tau)$ to derive the precipitation amounts P at any location and/or at any time, given that $S(\tau)$ is known. This is why such an estimation method has received attention over the last 15 years for estimating precipitation with remote sensing devices such as satellites or radars, see [1] to [12].

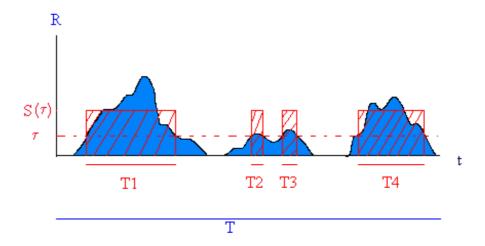


Figure 1: Geometrical interpretation of the threshold method.

The method estimates P, the area under the curve by P^* , the sum of the 4 squares. The value of $S(\tau)$ integrates information about the precipitation rates lower than τ .

$$P = \int_{t} Rdt$$

$$P^{*} = \int_{t} S(\tau)I(\tau)dt$$

$$P^{*} = S(\tau)\int_{t} I(\tau)dt$$

$$T(\tau) = \int_{t} I(\tau)dt = T1 + T2 + T3 + T4$$

$$P^{*} = S(\tau)T(\tau)$$

$$FTR(\tau) = \frac{T(\tau)}{T}$$

$$P^{*} = TS(\tau)FTR(\tau)$$

$$P^{*} = \beta(\tau)FTR(\tau)$$

III) Evaluation of the threshold method in Iceland

We propose now to assess the linearity of (15) over Iceland, considering monthly precipitation and annual precipitation, and thresholds of 0, 1, 5 and 10 mm/day.

The raw data used in this study are daily precipitation measured at 122 sites from 1980 to 2001 (figure 2). These precipitation stations are manual. No correction is applied to the data to account for any loss due to wind for instance. From these daily precipitation data, monthly and annual precipitations are derived. The monthly precipitation is computed if the station has been in operation for at least 25 days, and the annual precipitation is computed if the station has been in operation for at least 350 days. The number of stations used simultaneously in a given month is ranging from 60 to 122 with a mean of 84.

Figure 3 to 6 present for each month the linear regressions between $FTR(\tau)$ and P for the monthly precipitation in 2001 and figure 7 for the annual precipitation in 2001. The annual relationships derived from 1980 to 2000 are given in appendix 1 and the monthly relationships for the thresholds 5 mm/day and 10 mm/day are given in appendices 2 and 3. The statistics of the correlation coefficients are given in tables 1 and 2.

Table 1: monthly regressions: statistics of the correlation coefficients

	minimum	mean	maximum
0 mm/day	-0.259	0.409	0.795
1 mm/day	0.141	0.699	0.904
5 mm/day	0.600	0.869	0.956
10 mm/day	0.627	0.899	0.967

Table 2: annual regressions: statistics of the correlation coefficients

	minimum	mean	maximum
0 mm/day	0.105	0.359	0.574
1 mm/day	0.741	0.802	0.855
5 mm/day	0.913	0.940	0.962
10 mm/day	0.961	0.978	0.986

Figure 2: precipitation network

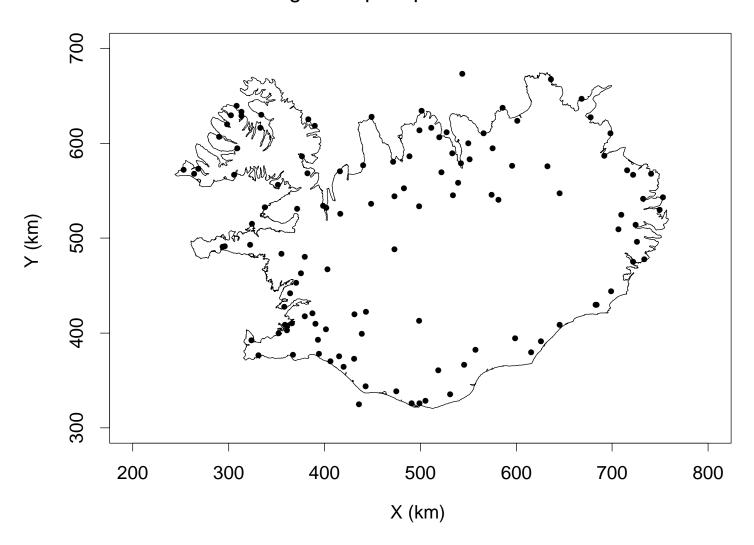
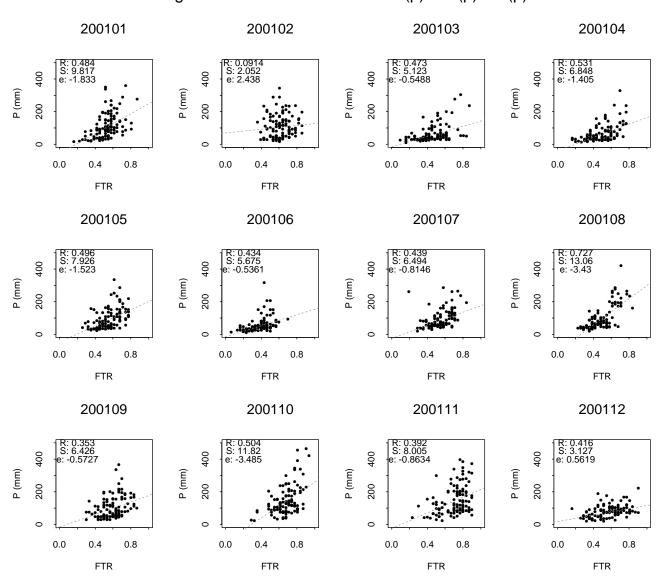


Figure 3: threshold=0mm: P=TS(p)FTR(p)+Te(p)



200101 200102 200103 200104 400 400 400 400 P (mm) P (mm) P (mm) P (mm) 200 200 200 200 0.4 0.8 0.0 0.4 8.0 0.0 0.8 0.0 0.0 0.4 0.8 0.4 FTR FTR FTR FTR 200105 200106 200107 200108 400 400 400 400 P (mm) P (mm) P (mm) P (mm) 200 200 200 200 0 0.8 0.8 0.4 0.0 8.0 0.0 0.4 0.0 0.4 0.0 0.8 0.4 FTR FTR FTR FTR 200110 200109 200111 200112

400

200

0.0

P (mm)

400

0.0

0.8

FTR

P (mm) 200 4

0.8

FTR

400

200

0

0.0

0.4

FTR

0.8

P (mm)

8.0

FTR

400

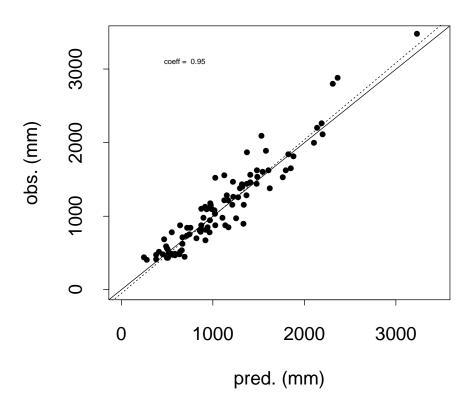
200

0.0

P (mm)

Figure 4: threshold=1mm: P=TS(p)FTR(p)+Te(p)

Figure 5: Annual precipitation in 2001



200103 400 400 400 400 P (mm) P (mm) P (mm) 200 200 200 200 0 0.8 0.0 0.4 8.0 0.0 0.4 0.8 0.0 0.4 0.0 0.4 0.8 FTR FTR FTR FTR 200105 200106 200107 200108 400 400 400 400 P (mm) P (mm) P (mm) P (mm) 200 200 200 200 0 0.0 0.8 0.0 0.4 8.0 0.4 0.8 0.0 0.4 0.0 0.4 0.8 FTR FTR FTR FTR 200109 200110 200111 200112

400

200

0.0

P (mm)

Figure 6: threshold=10mm: P=TS(p)FTR(p)+Te(p)

200104

R: 0.838 S: 19.11 e: 1.339

400

200

0.0

0.4

FTR

0.8

P (mm)

0.8

FTR

200102

400

200

0

0.0

0.4

FTR

0.8

P (mm)

200101

400

200

0.0

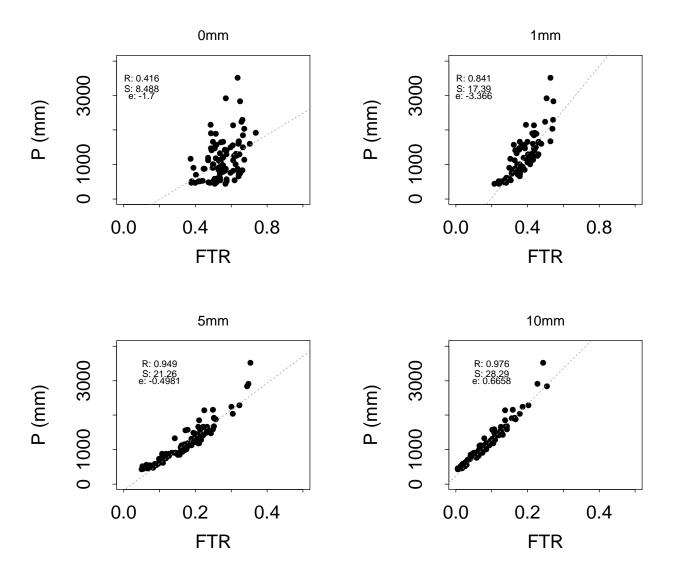
0.4

FTR

8.0

P (mm)

Figure 7: Year 2001 : P=TS(p)FTR(p)+Te(p)



One can see that the linearity of (15) is very poor for the thresholds 0 and 1 mm/day, becomes much more acceptable for 5 mm/day and reaches an optimum (highest correlation coefficient) at $\tau = 10$ mm/day for both monthly and annual precipitation. The poor linearity observed at $\tau=0$ mm/day and 1 mm/day shows that the probability distribution function (pdf) of the strictly positive precipitation rate is probably not stationary in space and time. At the optimal threshold (10 mm/day) the observed linearity of the relationship between $FTR(\tau)$ and P suggests that $S(\tau)$ can be considered reasonably stationary in space (over Iceland) for a given month or year despite changes in the pdf of rain rate. Thus, the estimation of monthly and/or annual precipitation at any location in Iceland can be performed reasonably well by simply counting the number of days exceeding 10 mm. At this optimum threshold of 10 mm/day, $FTR(\tau)$ explains in average 80% of the variance of the monthly precipitation and 95% of the annual precipitation. In other words, with respect to the data available for this study, the spatial distribution of precipitation in Iceland is rather well explained by the spatial distribution of the number of days exceeding 10 mm/day.

All the linear relationships present an intercept term. This intercept term accounts for the precipitation lower than the threshold and not detected by the method. This term is usually rather low for both $\tau = 5$ mm/day and 10 mm/day.

A the optimal threshold, the correlation coefficients derived from the annual relationships are higher than those of the monthly correlations. There are two reasons at least that may explain this fact:

- i) The temporal resolution of the data with respect to the length of the period. With daily precipitation, $FTR(\tau)$ is more accurately estimated over one year than one month, and this has an effect on the linearity of the relationship.
- ii) The spatial variability of the precipitation rates generated within one month is more important than within one year, and this is affecting directly the stability of $S(\tau)$ and the strength of the linear relationship (15). There is a lower limit in the use of the method related to the length of the time window.

Figure 8 presents the interannual variability of $S(\tau)$ derived from the annual linear regressions, for the thresholds 5 and 10 mm/day, and the mean value over Iceland of E[R|R>0]. A climatological analysis of these relationships is outside the scope of this study. However, it is worth noting that a trend is observed for the threshold 5 mm/day (and to some extent 10 mm/day) showing that in average over Iceland, the slope of the annual linear relationships has apparently slowly increased over the last 20 years. This observed trend might be a reality or simply a pure sampling effect due to a change in the spatial distribution of the precipitation network (the number of stations used in the study has almost double from 1980 to 2000). Whatever the reasons involved, there has been a slight change in the experimental pdf of R>0, causing either an increase in E[R|R>0] and/or a reduction of $Pr(R > \tau | R > 0)$. The assumption of a slight increase in E[R|R>0] at least seems to be supported. On the other hand, the interannual variability of the annual $FTR(\tau)$ does not present any marked trend (see figures 9 and 10).

The interannual variability of $S(\tau)$ derived from the monthly linear regressions is presented in figure 11 for the threshold 10 mm/day only. The slopes derived from the other thresholds are not considered because of high uncertainty due to the too poor linear relationships between $FTR(\tau)$ and P. One can see that for some specific months $S(\tau)$ presents a trend describing an increase of $S(\tau)$ like January, June, September and December, while other months present a trend describing a decrease of $S(\tau)$ like February and April. A detailed analysis of the pdf's of R should be carried out with an homogeneous precipitation network in order to understand how significant these results are, and describe the long term evolution of the precipitation rates over Iceland.

Despite the observed interannual variability of the $S(\tau)$ coefficients, the threshold method still works remarkably well when all the monthly data of each month respectively are lumped together, (figure 12) or when all the monthly data of each year are lumped together (figures 13 and 14).

Figure 8: Interannual variability of annual S(p) 5 mm/day

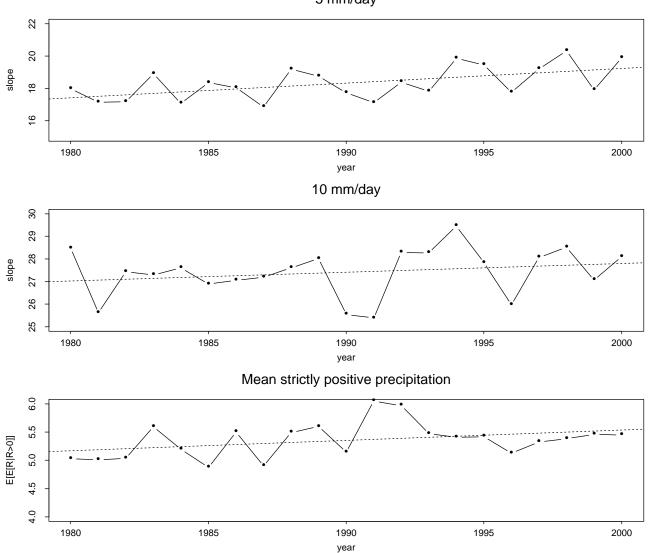


Figure 9: Interannual variability of annual FTR(5mm/day)

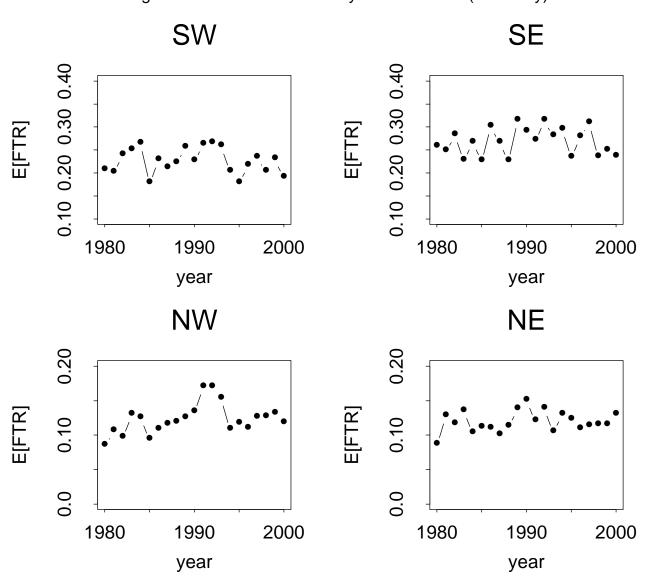
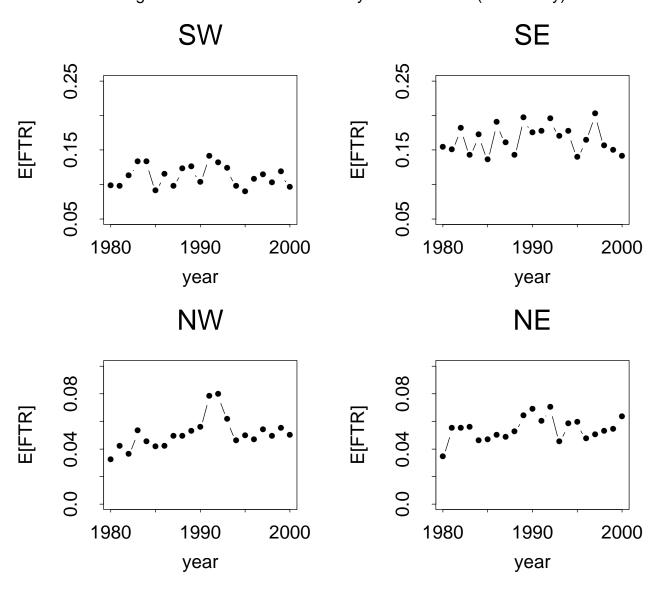
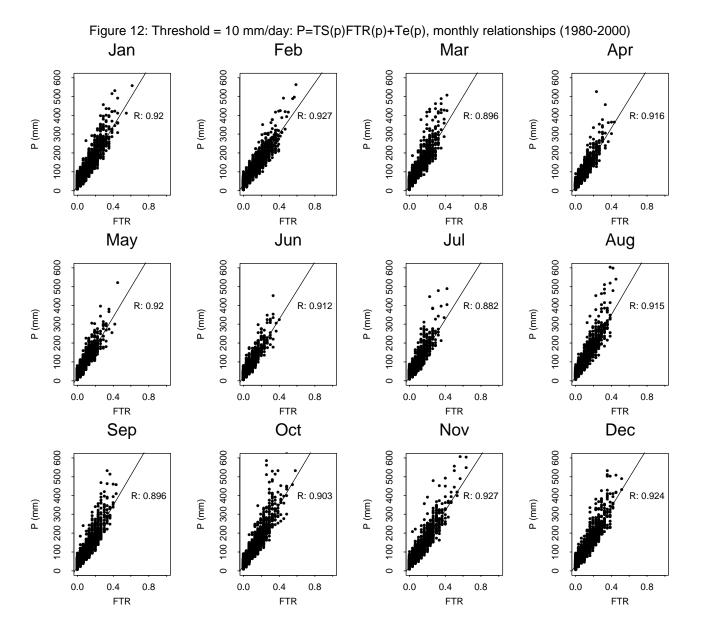


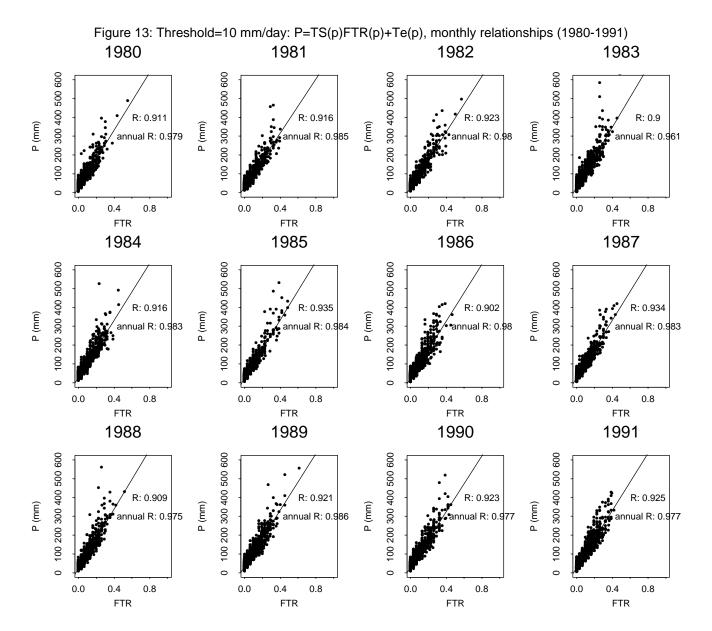
Figure 10: Interannual variability of annual FTR(10mm/day)

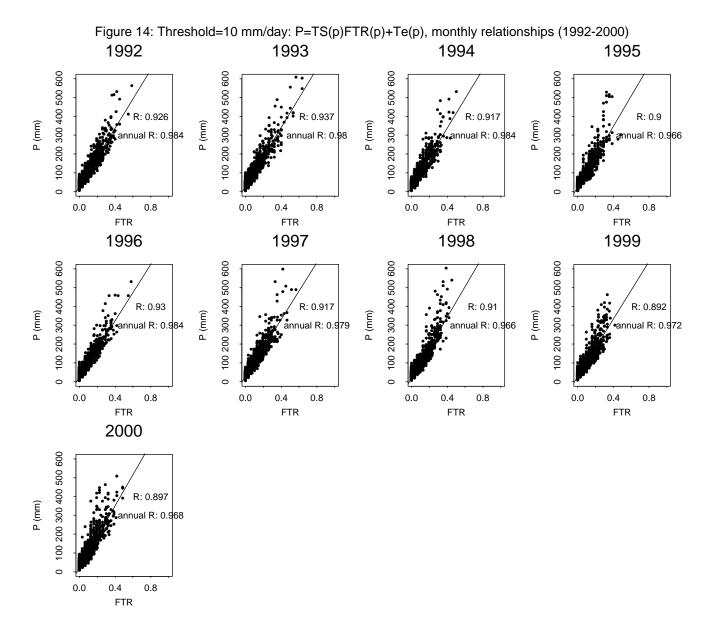


Jan Feb Mar Apr slope slope slope year year year year May Jun Jul Aug slope slope slope slope year year year year Oct Sep Nov Dec slope slope slope slope year year year year

Figure 11: Interannual variability of monthly S(10mm/day)







IV) Conclusion

The purpose of this short study was to analyze the relationships between precipitation and intermittency in Iceland. With respect to the data resolution (daily precipitation). it was observed that the number of days exceeding 10 mm of precipitation explains in average 80% of the variance of monthly precipitation and 95% of the variance of annual precipitation in Iceland. The reason for that is the stability of the coefficient $S(\tau)$. These results open several perspectives. The proposed method can be usefull to provide insight on long term variability of precipitation in Iceland with few meaningfull parameters. It is also suitable for dealing with the estimation of precipitation in Iceland with the use of remote sensing devices. In the near future, these results will be used as reference in the validation protocol developed for assessing the quality of the precipitation amounts produced by the MM5 numerical model in Iceland [13] and [14]. It will help to identify which aspect of the precipitation process is (or is not) reproduced with satisfaction by this model, and understand if the differences in the precipitation patterns/structure are caused by differences in the length of time precipitation is occurring and/or differences in the precipitation rates.

Acknowledgements

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References

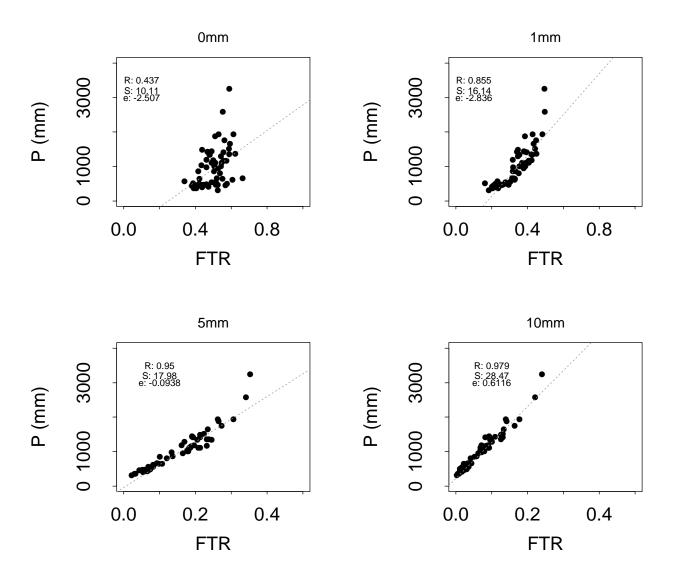
- [1] Atlas, I., Rosenfeld, D., Short, D.A., 1990. Estimation of convective rainfall by area integrals 1. Theoretical and empirical basis. J. Geophys. Res. 95(D3), 2153-2160.
- [2] Atlas, D., Bell, T.L., 1992. The relation of radar to cloud area-time integrals and implications for rain measurements from space. mon. Wea. Rev. 120, 1997-2008.
- [3] Braud, I., Creutin, J.D., Barancourt, C., 1993. The relation between the mean areal rainfall and the fractional area where it rains above a given threshold. J. Appl. Meteorol. 32, 193-202.
- [4] Crochet, P., 1999. Evaluation of the threshold methods for rainfall estimation over a boreal region. A case study based on the NOPEX radar data. Agric. For. Meteorol. (98-99), 349-362.
- [5] Kedem, B., Chiu, L.S., Karni, Z., 1990. An analysis of the threshold method for measuring area average rainfall. J. Appl. Meteorol. 29, 3-20
- [6] Kedem, B., Chiu, L.S., North, G.R., 1990. Estimation of the mean rain rate: Application to satellite observations. J. Geophys. res. 95(D2), 1965-1972.
- [7] Kedem, B., Pavlopoulos, H., 1991. On the threshold method for rainfall estimation: Choosing the optimal threshold level. J. Am. Stat. Assoc. 86, 626-633.
- [8] Morrissey, M.L., 1994. The effect of data resolution on the area threshold method. J. Appl. Meteorol. 33, 1263-1270.
- [9] Morrissey, M.L., Krajewski, W.F., Mc Phaden, M.S., 1994. Estimating rainfall in the tropics using the fractional time raining. J. Appl. Meteorol. 33, 387-393.
- [10] Rosenfeld, D., Atlas, D., Short, D.A., 1990. The estimation of convective rainfall by area integrals 2. The height-area rainfall threshold (HART) method. J. Geophys. Res. 95(D3), 2153-2160.
- [11] Short, D.A., Wolf, D.B., Rosenfeld, D., Atlas, D., 1993. A study of the threshold method utilizing raingauge data. J. Appl. Meteorol. 32, 1379-1387.
- [12] Short, D.A., Shimizu, K., Kedem, B., 1993. Optimal thresholds for the estimation of area rain rate moments by the threshold method. J. Appl. Meteorol. 32, 182-192.
- [13] Rögnvaldsson, Ó and H. Ólafsson, 2001: Validation of high-resolution simulations with the MM5 system. Proc. NCAR MM5 workshop, Boulder, USA, June 2002.
- [14] Ólafsson, H., Á. J. Elíasson og Egill Porsteins, 2002: Orographic influence on wet snow icing conditions, Part I: Upstream of mountains. Proc. Intern. Workshop on Atmos. Icing on Structures, Prag, Czech Republic, June 2002.

Appendix 1

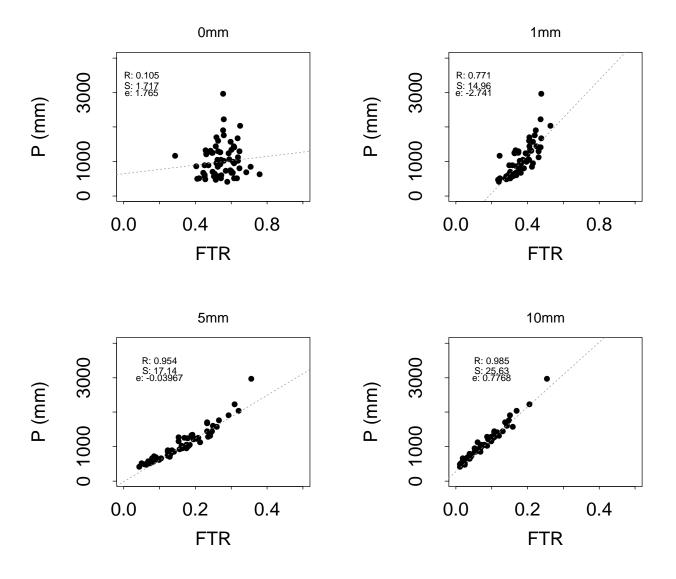
Relationship between annual precipitation and fractional time raining above a threshold

Period 1980-2000

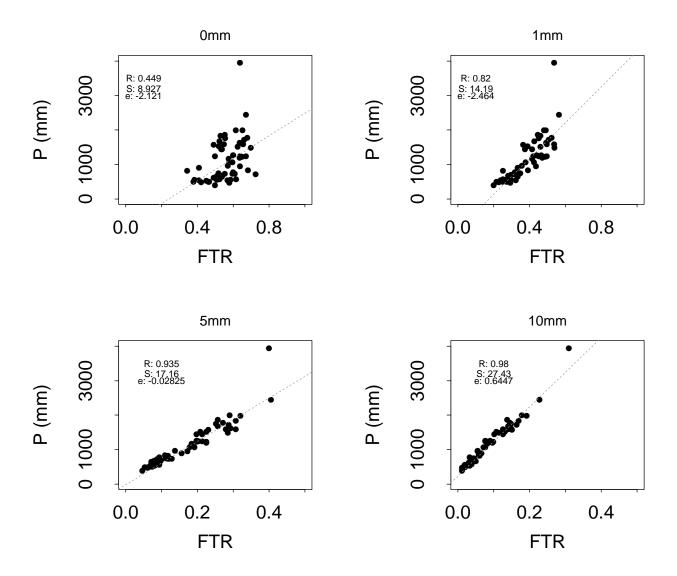
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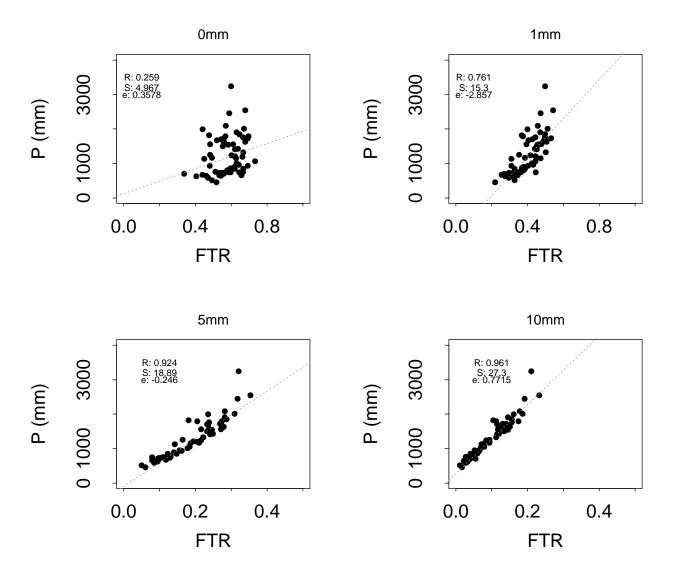
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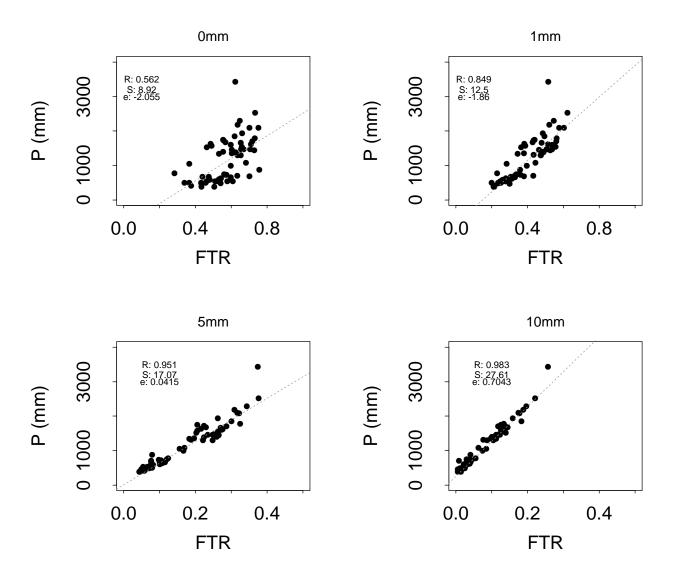
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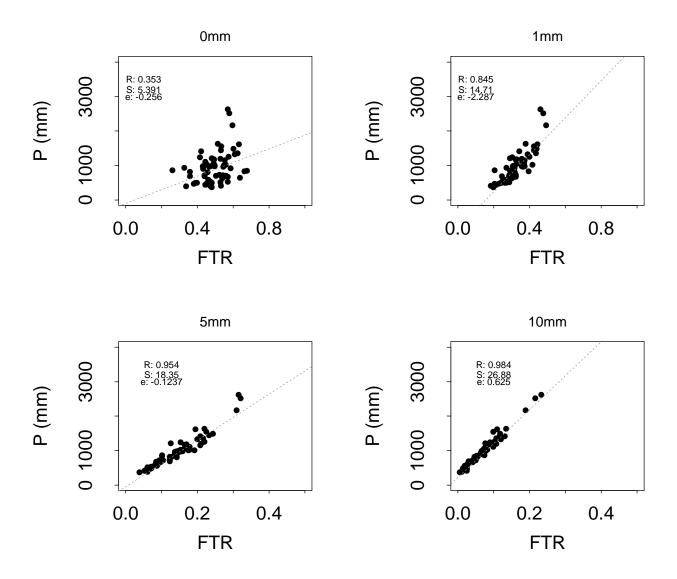
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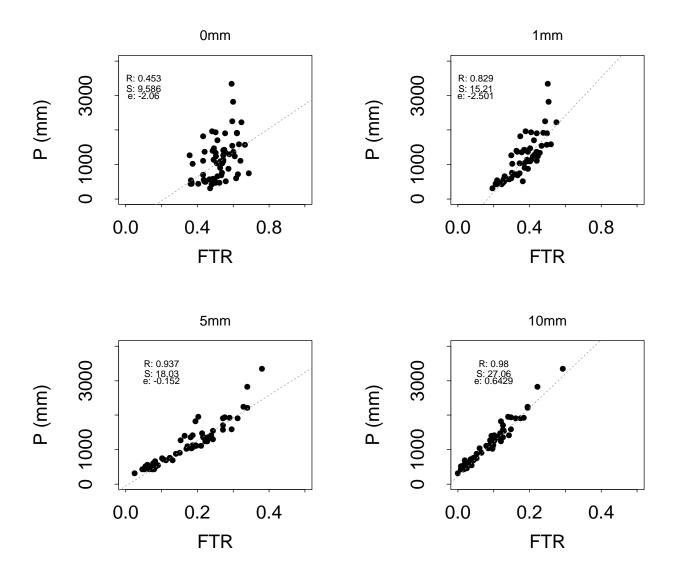
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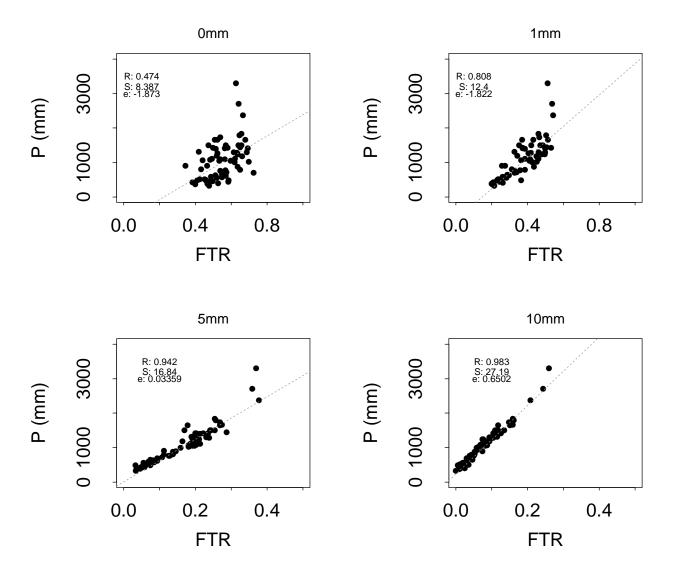
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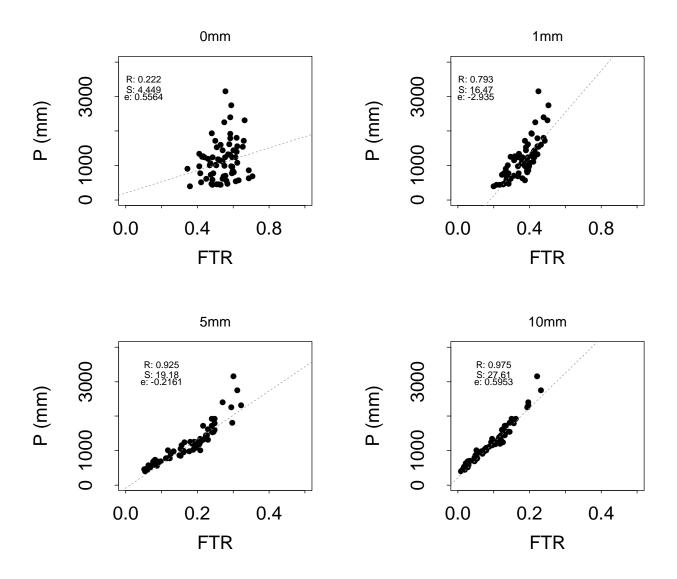
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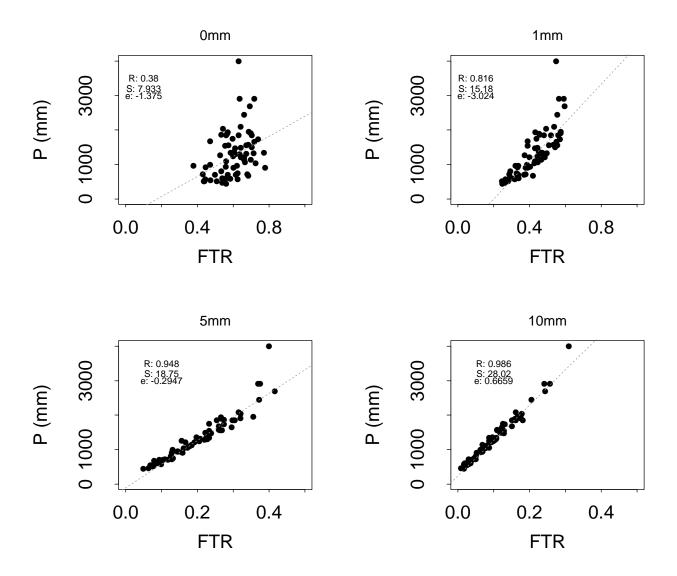
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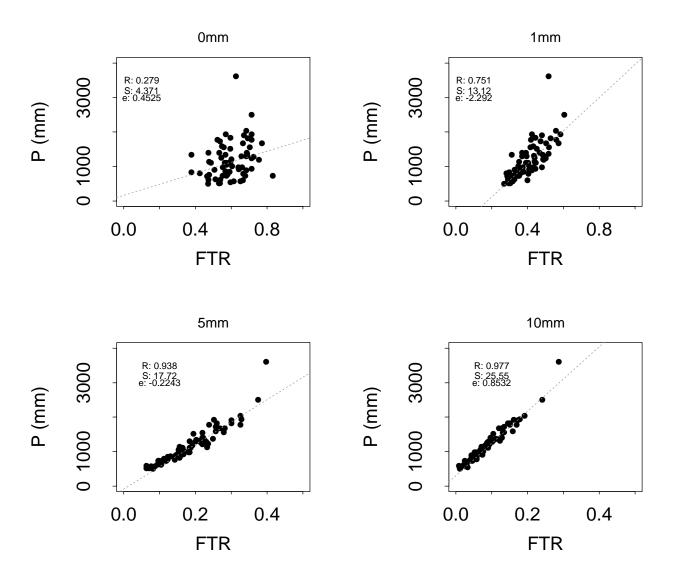
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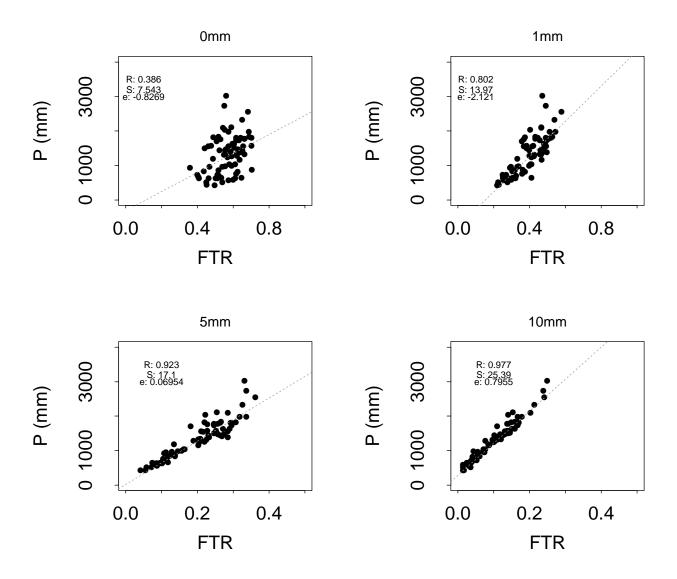
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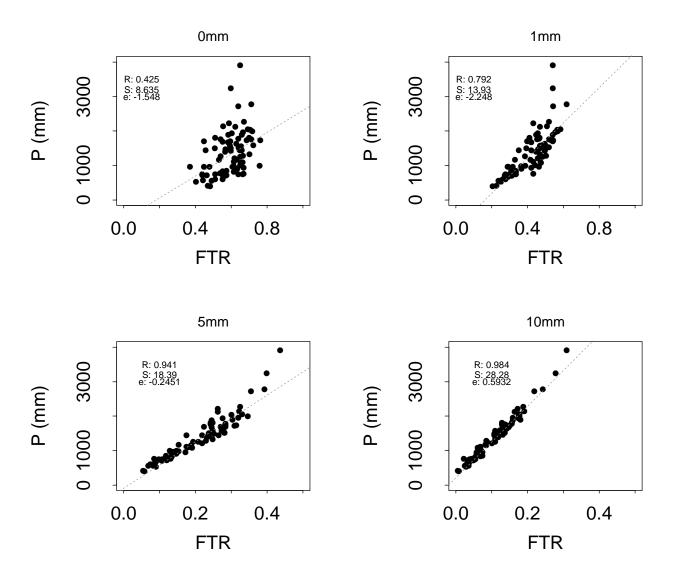
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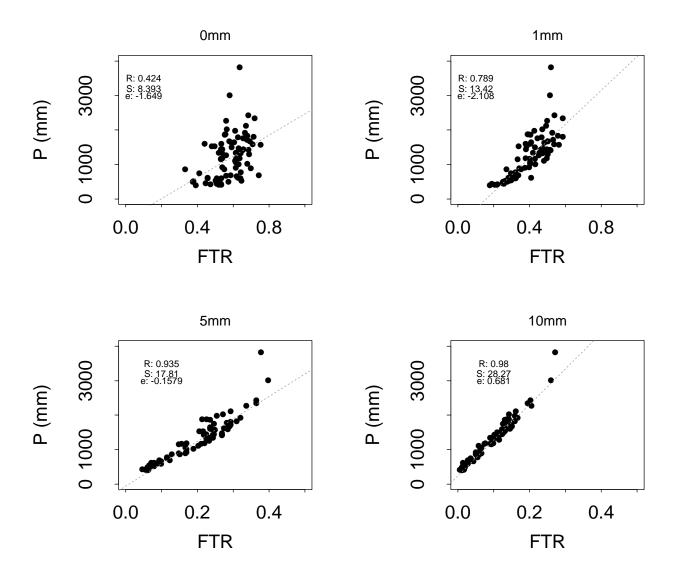
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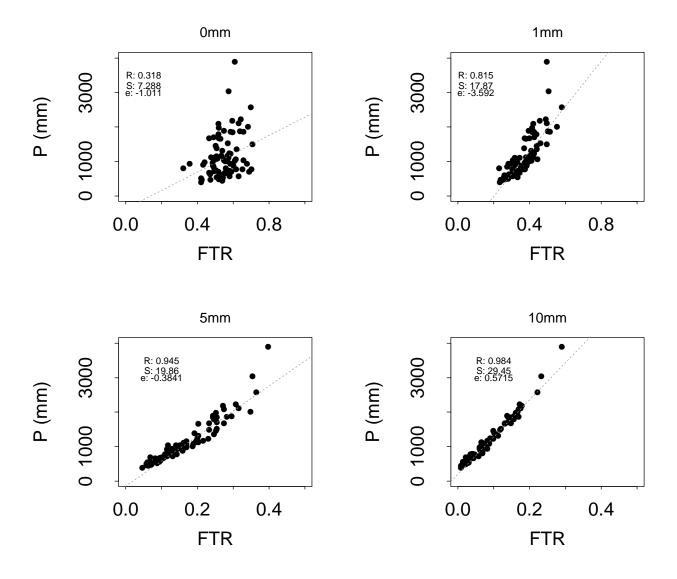
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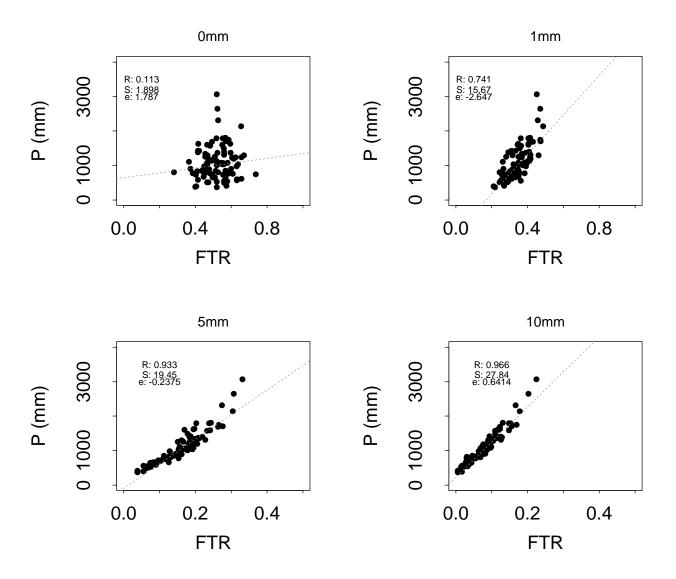
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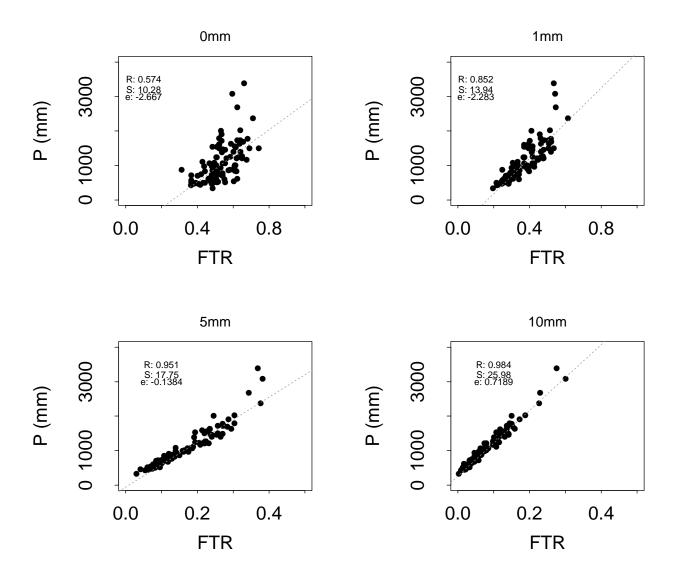
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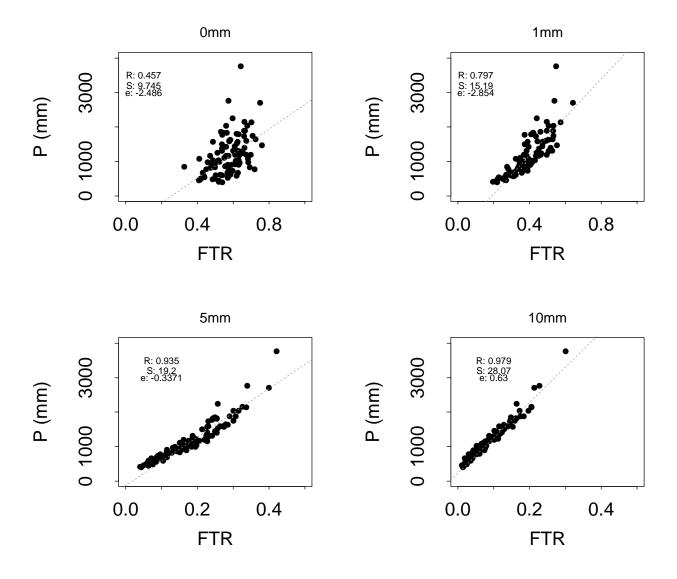
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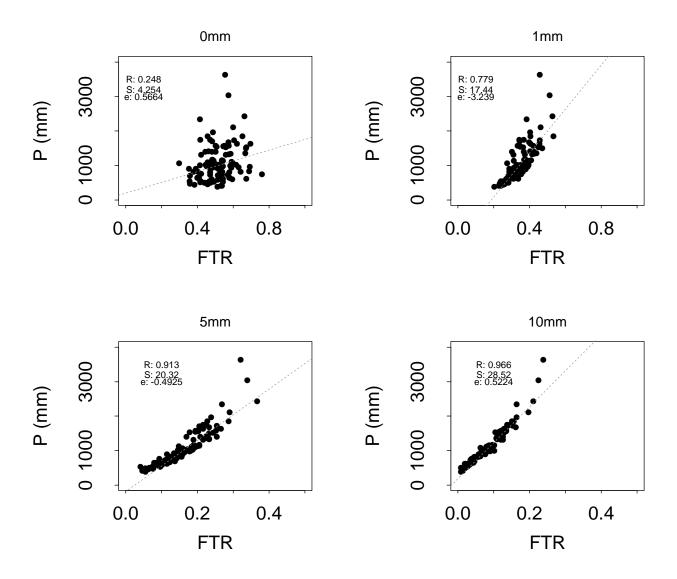
1996 : P=TS(p)FTR(p)+Te(p)



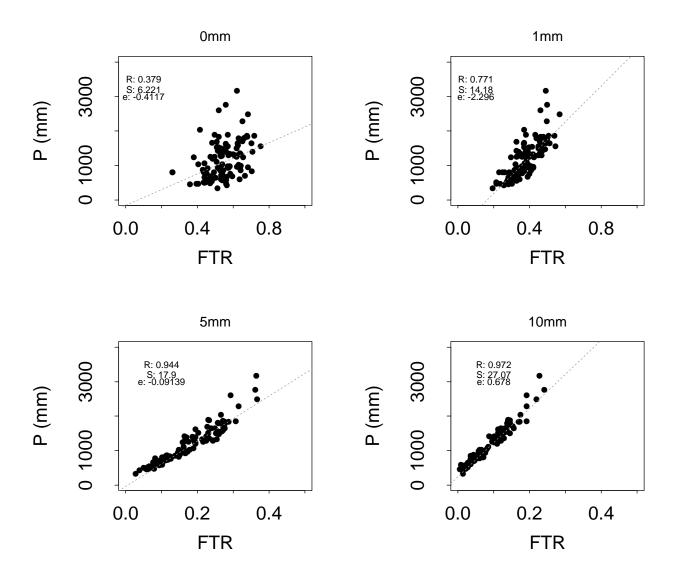
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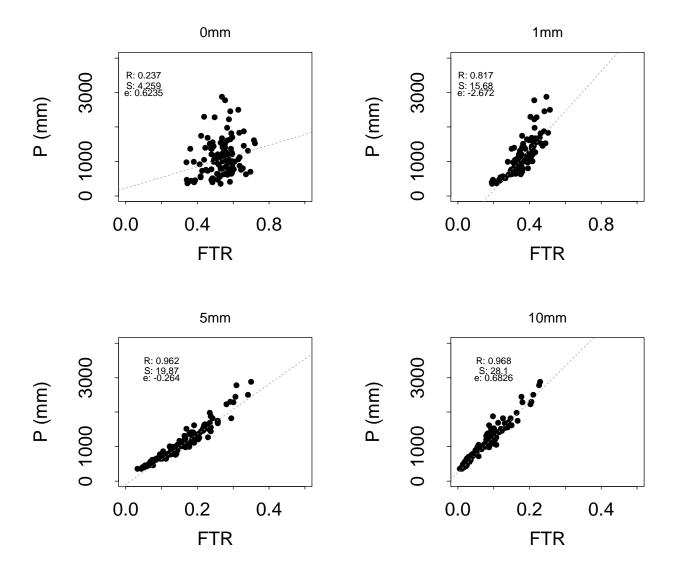
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1999 : P=TS(p)FTR(p)+Te(p)



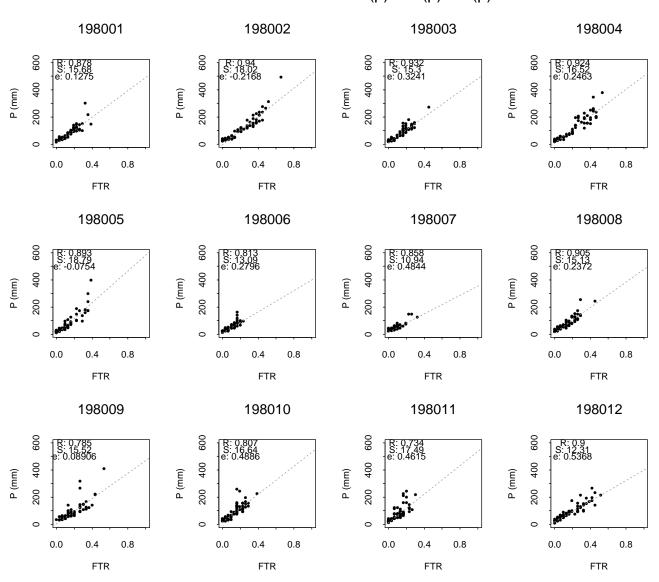
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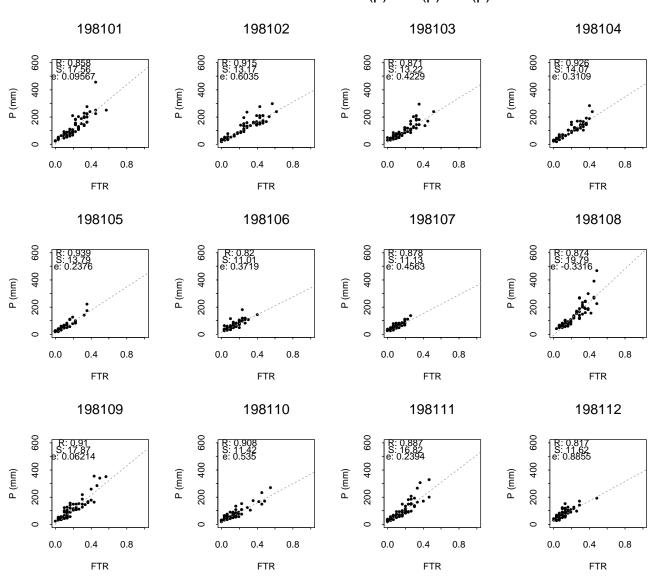


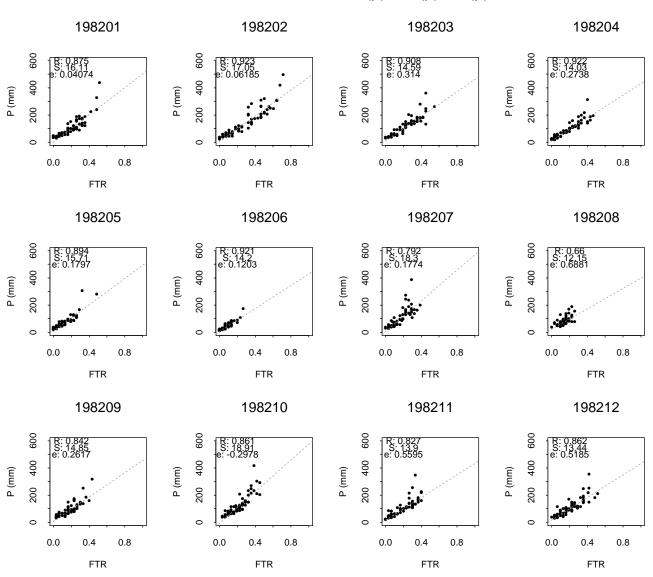
Appendix 2

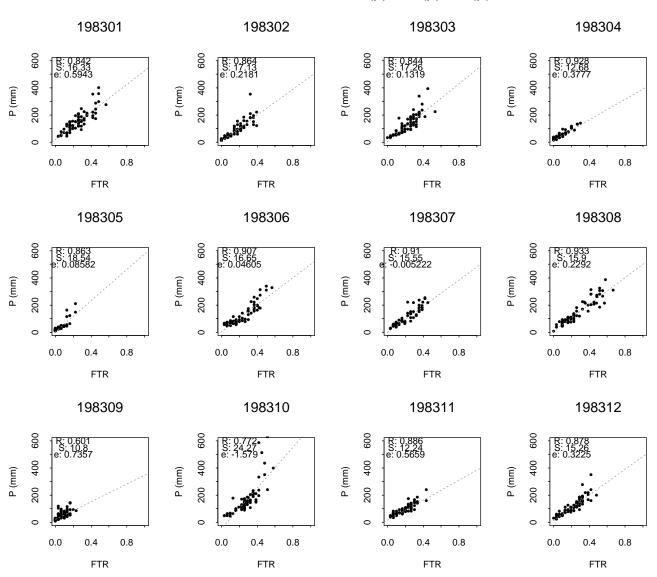
Relationship between monthly precipitation and fractional time raining above 5 mm/day

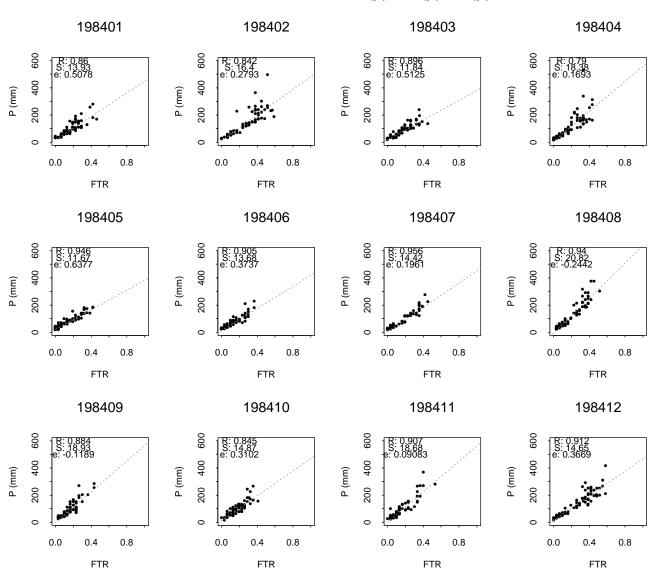
Period 1980-2000

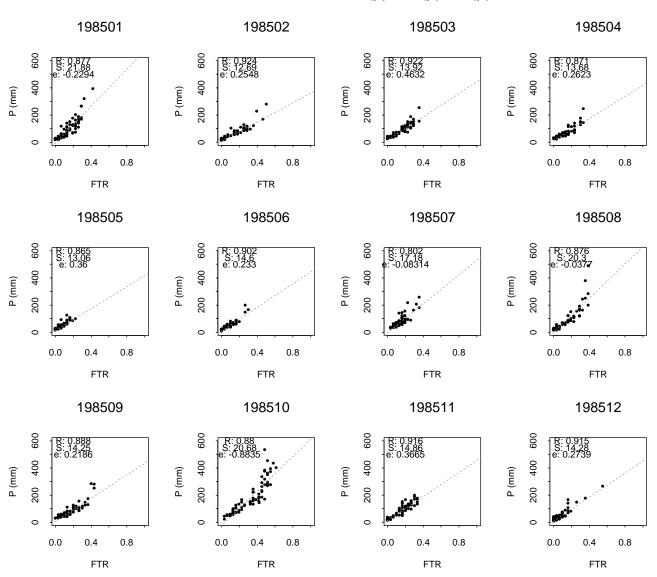


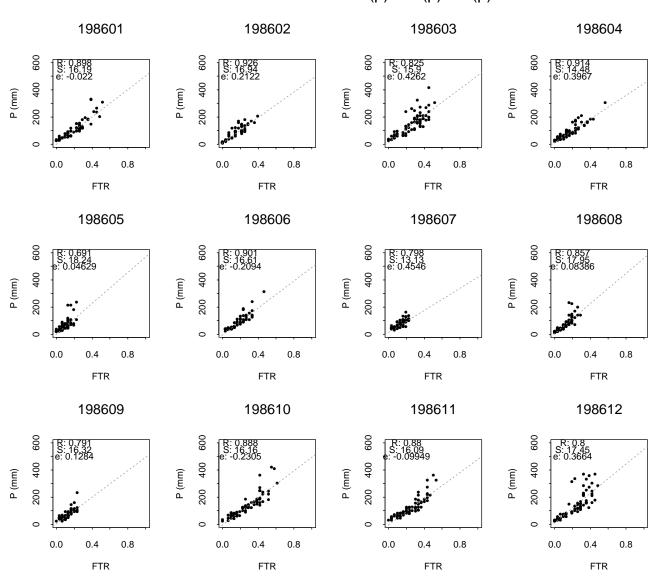


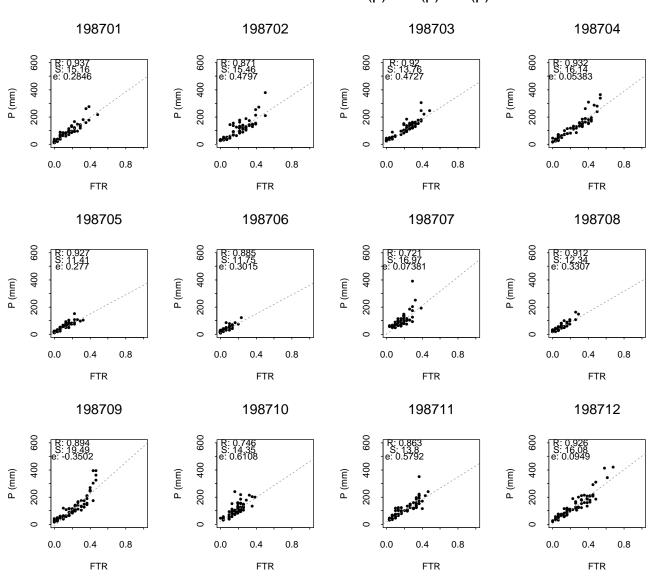


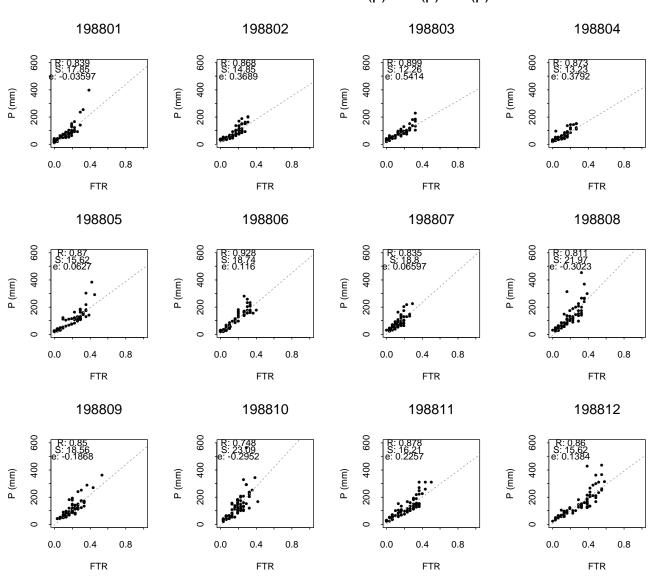


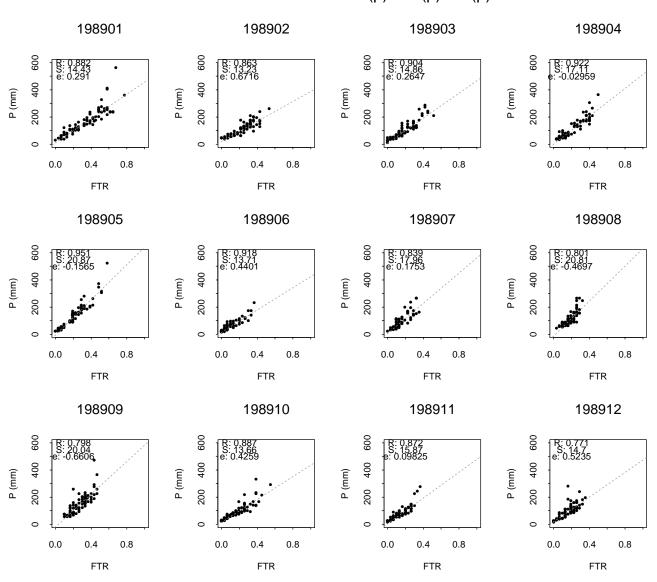


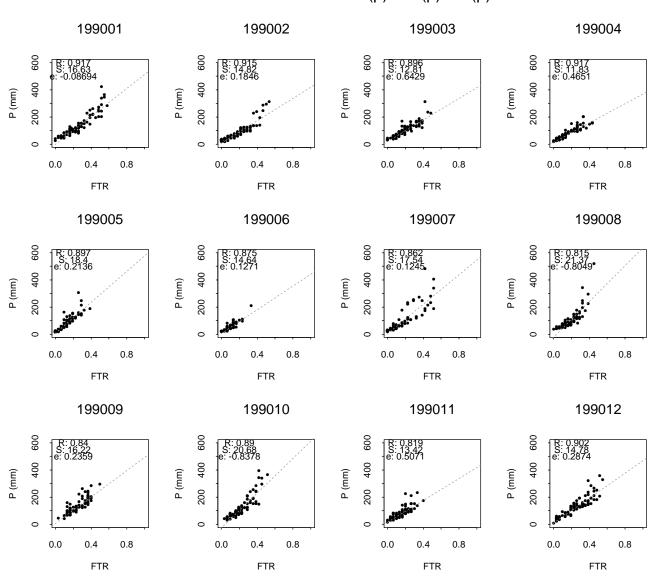


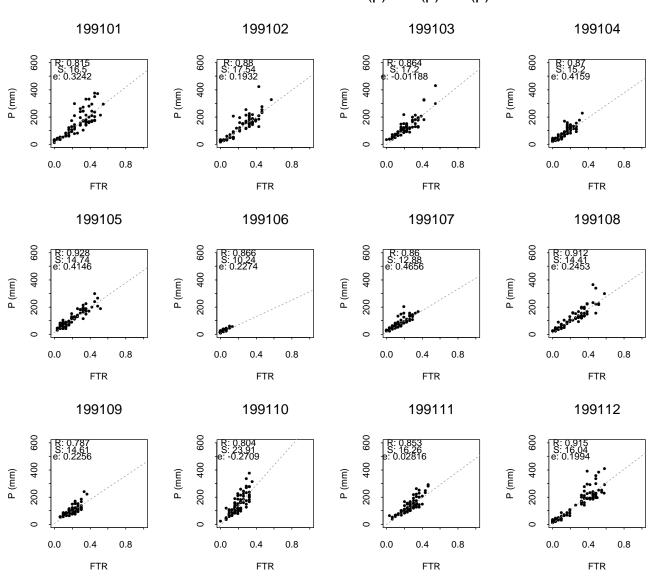


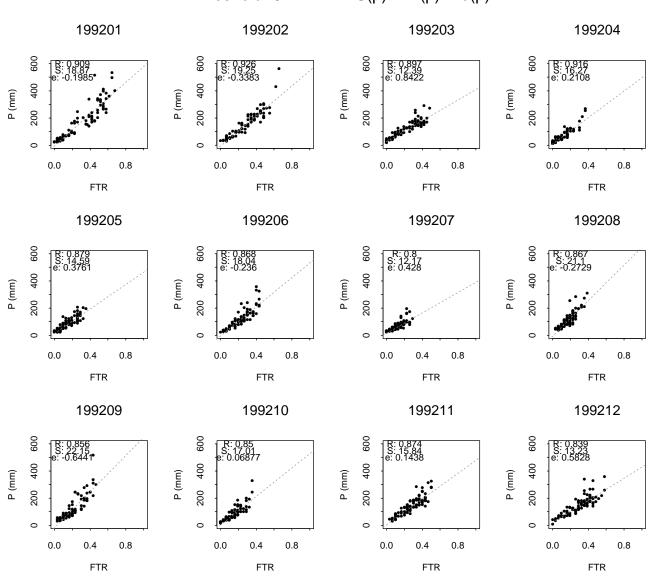


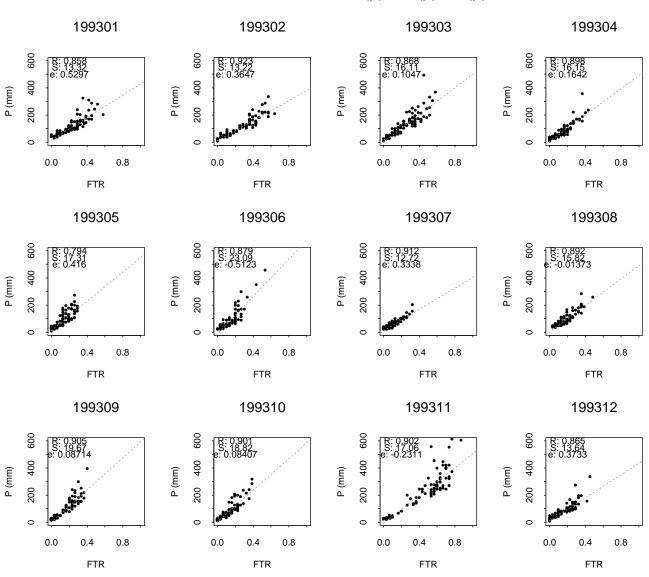


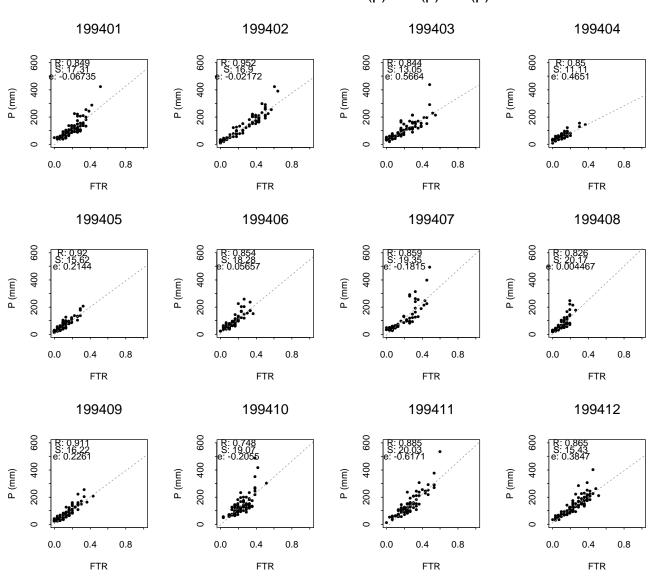


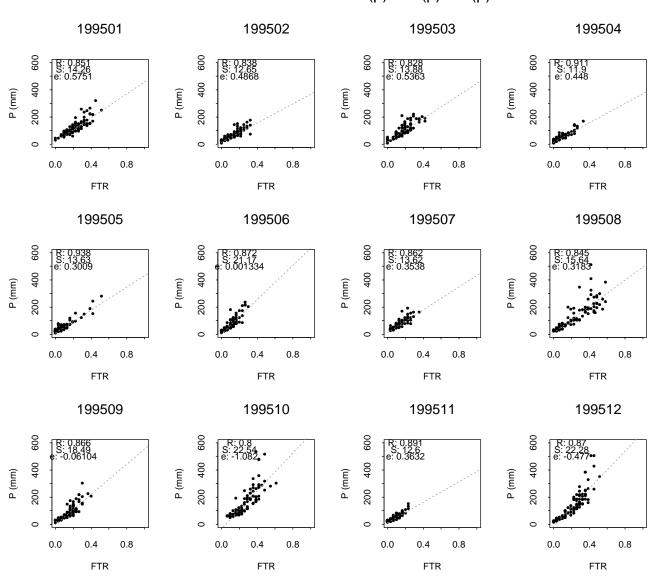


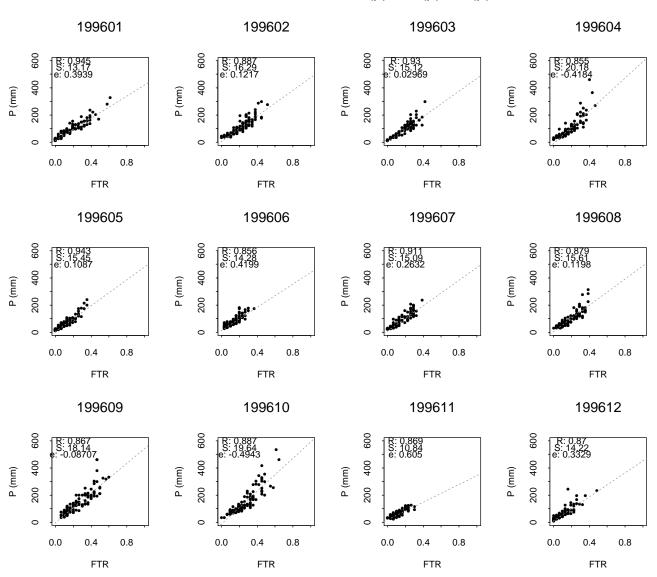


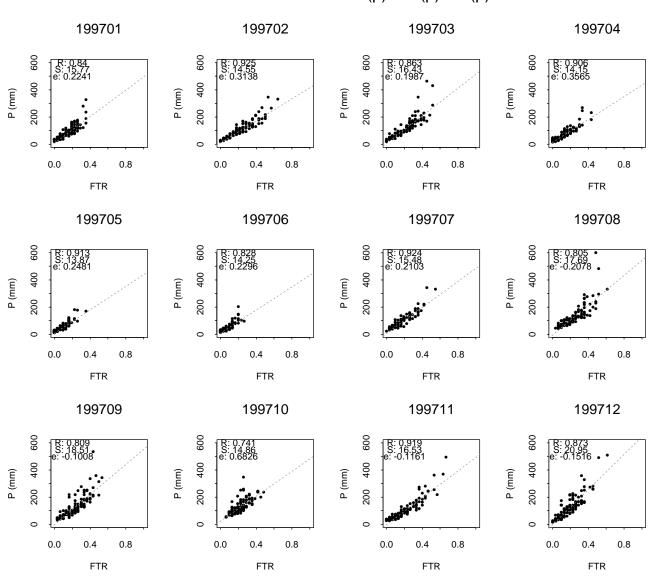


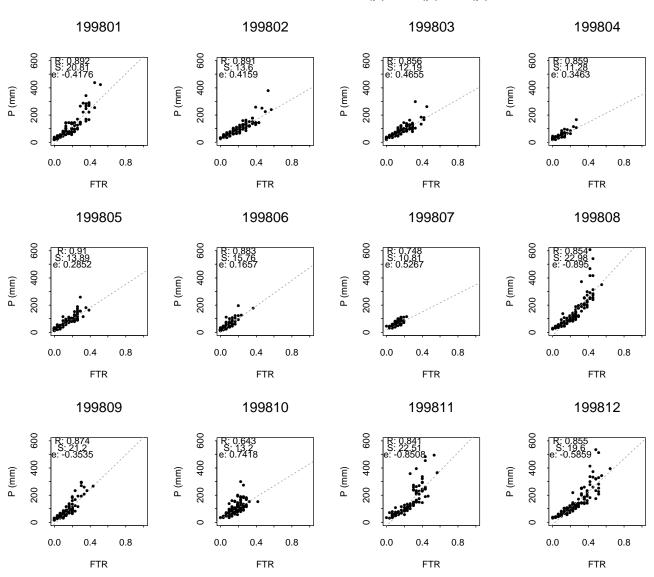


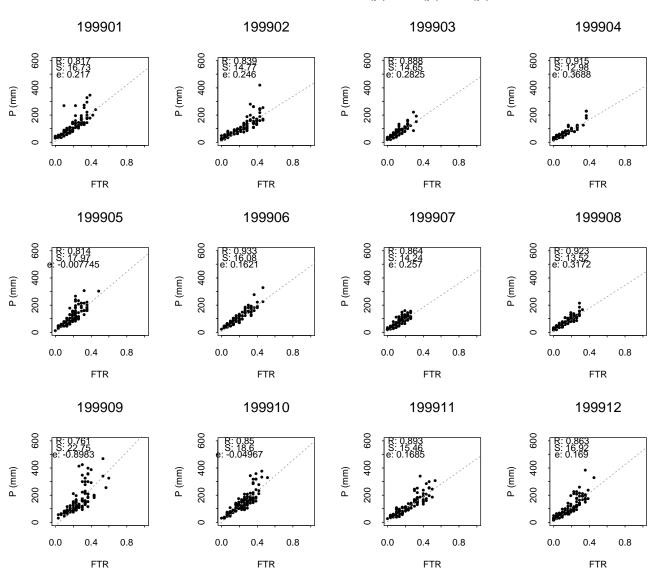


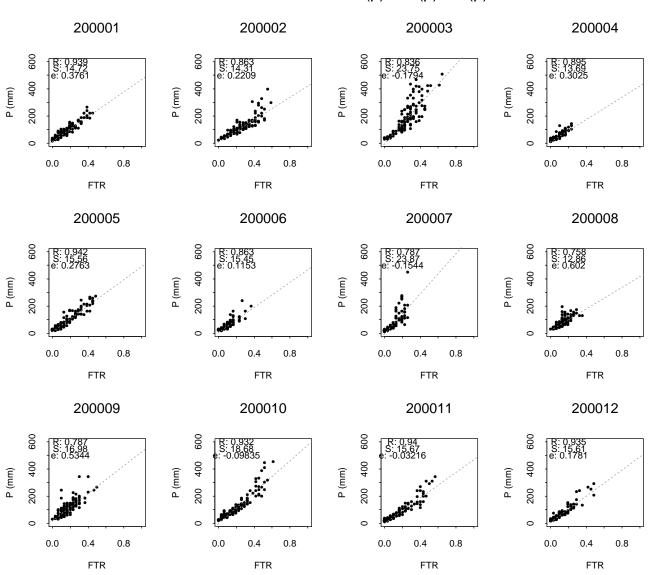












Appendix 3

Relationship between monthly precipitation and fractional time raining above 10 mm/day

Period 1980-2000

